Trigonometry Workbook

The unit circle

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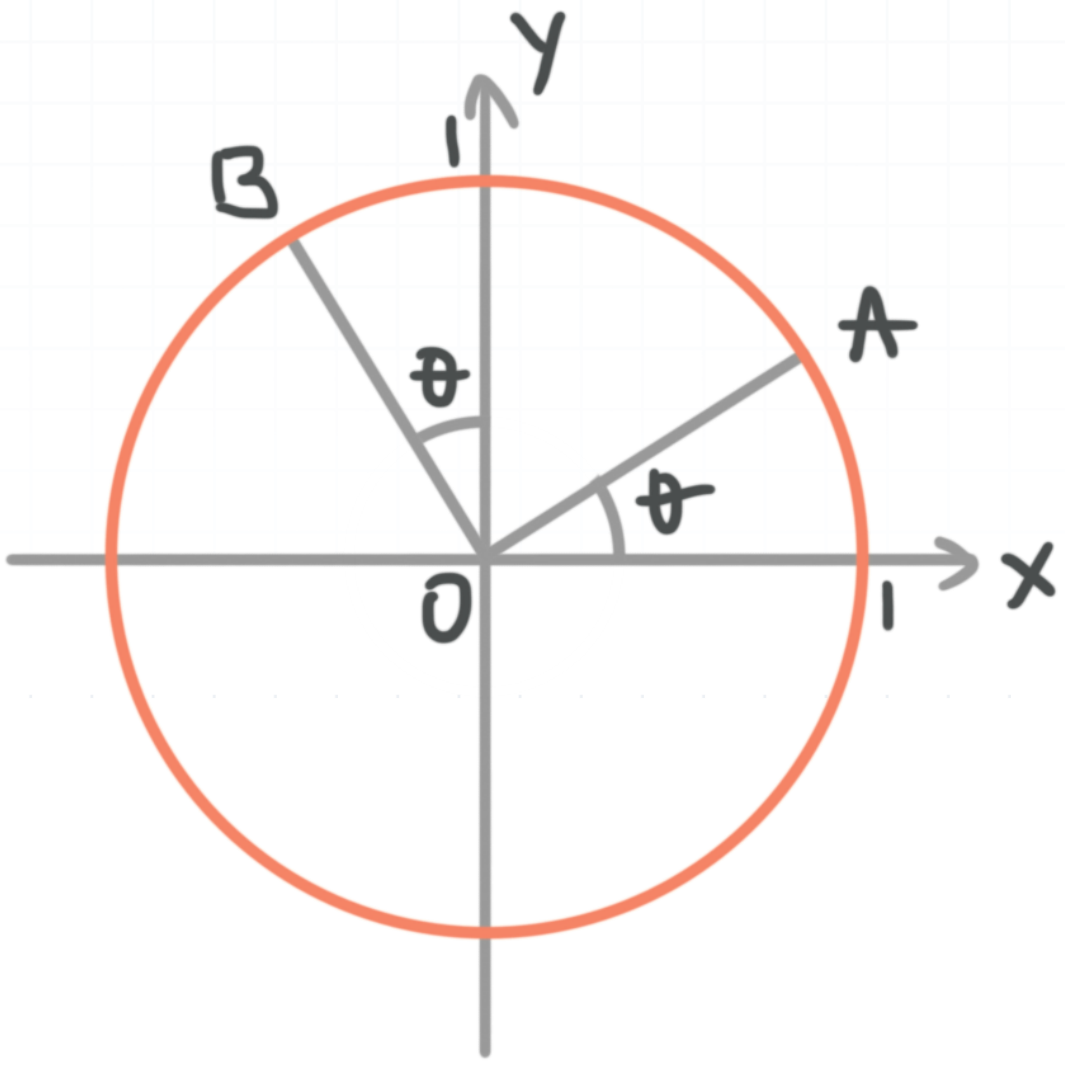
MATH

THE UNIT CIRCLE

■ 1. The points A and B lie on the unit circle in quadrants I and II respectively. The angle between OA and the positive direction of the x-axis is θ. The angle between OB and the positive direction of the y-axis is θ. In other words, these two angles have the same measure. Find the sine of angle AOB.



■ 2. Find the coordinates of a point P which is the point where the terminal side of the angle $\theta = 60°$ intersects the circle $x^2 + y^2 = 4$.

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3. For some angle $\theta \in [0,\pi/2)$, its sine and cosine have a ratio of $3:4$. Find the coordinates of a point P which is located where the terminal side of θ intersects the unit circle.

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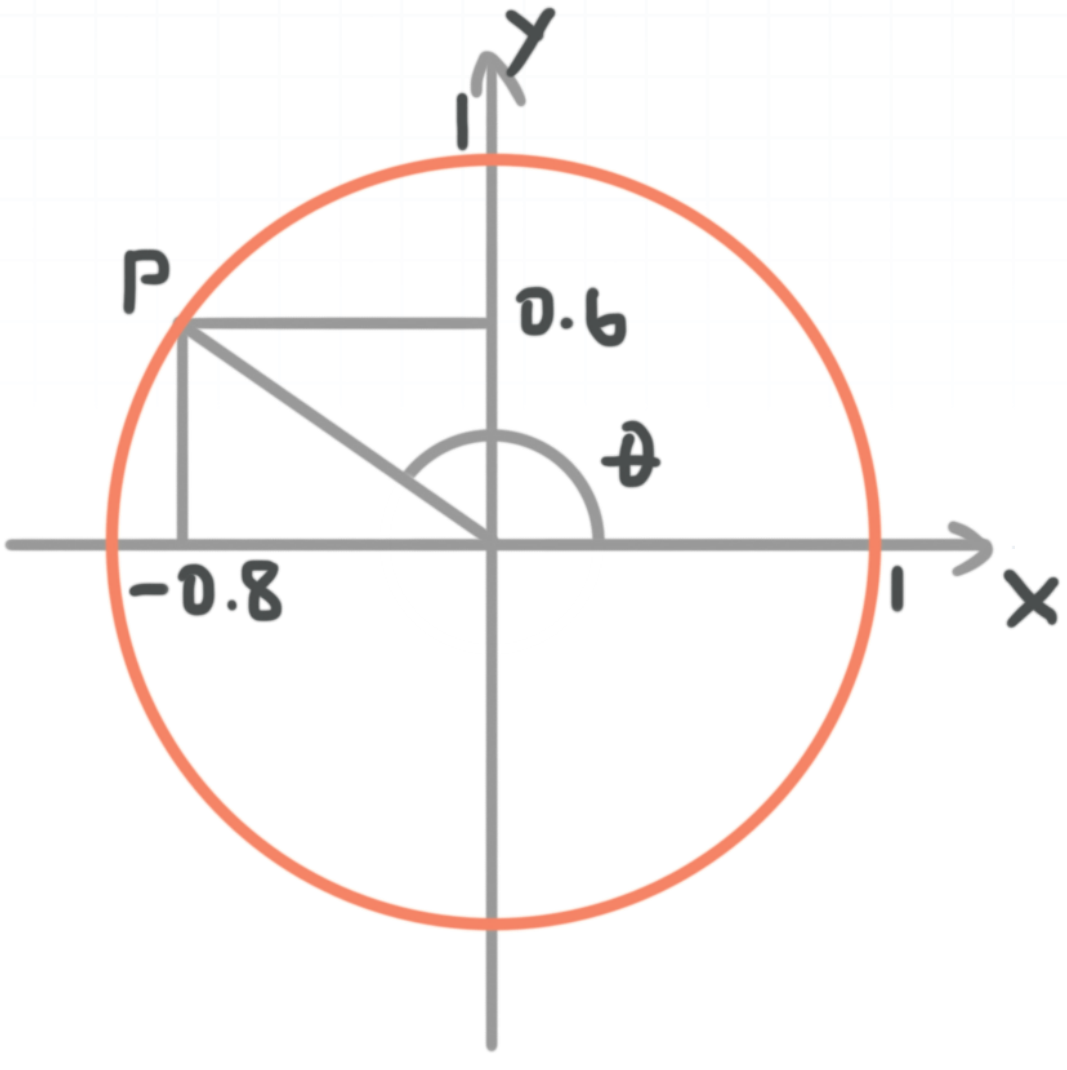
FINDING SINE AND COSINE OF AN ANGLE THETA

■ 1. Use the unit circle to find all possible values of θ.

$$|\cos\theta| = \cos 39^\circ$$

■ 2. The terminal side of the angle θ intersects the unit circle at $P(-0.8,0.6)$. Find the values of all six circular functions of θ.



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3. Consider the angles $10°$, $100°$, $190°$, and $280°$. Sort them in ascending order of their cosines.

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FINDING SINE GIVEN COSINE, OR VICE VERSA

■ 1. Find $\sin\theta$ if $\theta \in [0,180°]$. Hint: make a substitution into the Pythagorean identity $\sin^2\theta = 1 - \cos^2$.

$$\cos^2\theta - 0.36 = 0$$

■ 2. Find $\sin\theta$ and $\sin\phi$ if $\theta, \phi \in [0,2\pi)$.

$$\cos\theta + \cos\phi = 2$$

■ 3. Find $\sin\theta$ if $\theta \in [0,2\pi)$. Hint: make a substitution into the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$.

$$\cos\theta = \sin\theta$$

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ALL POSSIBLE VALUES FOR THE ANGLE THETA

■ 1. Consider the equation $f(x) = 0$, where $f(x)$ is some trigonometric function with period 2π. The equation $f(x) = 0$ has two solutions in $[0,2\pi)$, namely $\pi/4$ and $5\pi/4$. Extend these solutions to all of the periods and write them down using set notation in the simplest way possible.

■ 2. Solve the trigonometric equation.

$$\cos(3x + 5\pi) = 0$$

■ 3. Find the set of points on the xy-plane that satisfy the equation.

$$\sin x \cdot \cos y = 0$$

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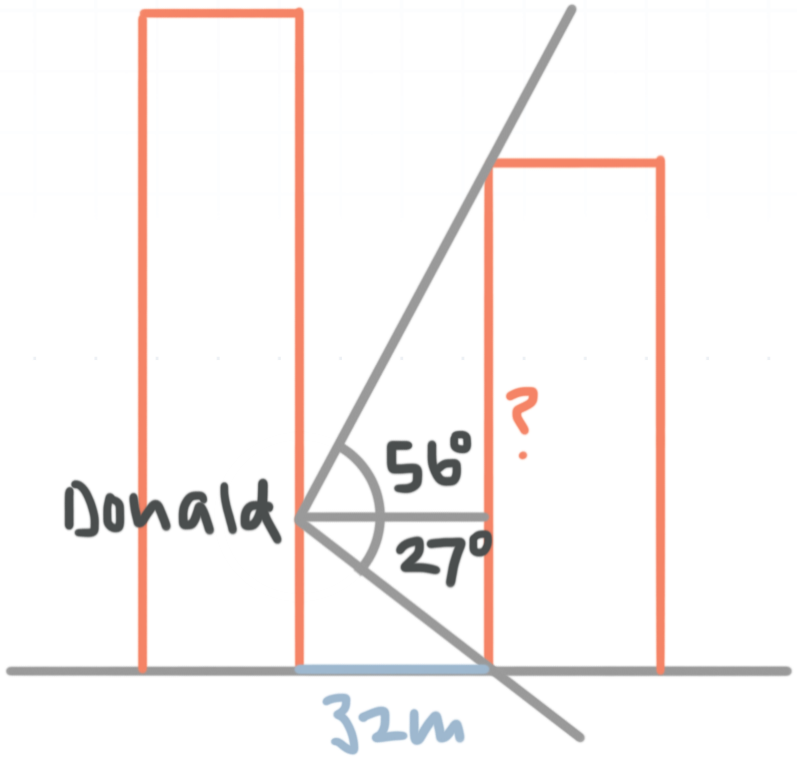
COMPLETING A RIGHT TRIANGLE

■ 1. Find the height of the isosceles triangle ABC if $AB = BC = 12$ and $m\angle B = 120°$.

■ 2. Donald looks from his window at a neighbor's house. Donald measures the elevation angle of the highest point of the house, $52°$, and the depression angle of the lowest point of the house, $27°$. Donald knows that the distance between the houses is 32 m. Find the height of the neighbor's house.



■ 3. A plot of land shaped like a right triangle has its hypotenuse along a river. The angle between one of the legs and the river is $30°$, and the distance from the river to the furthest point of the plot is 120 m.

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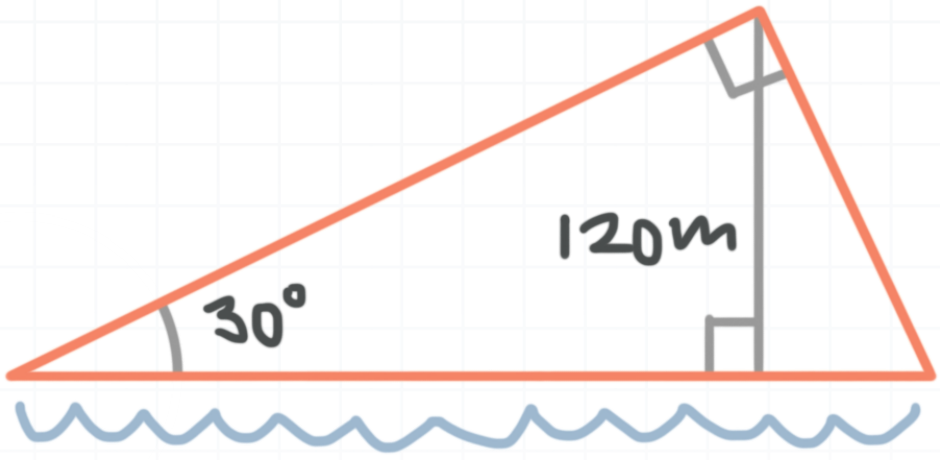
The land owner decides to build a fence around the plot and needs to find the total length of the fence. Since the fence is sold by sections of 10 meters, round up to the nearest 10 meters.



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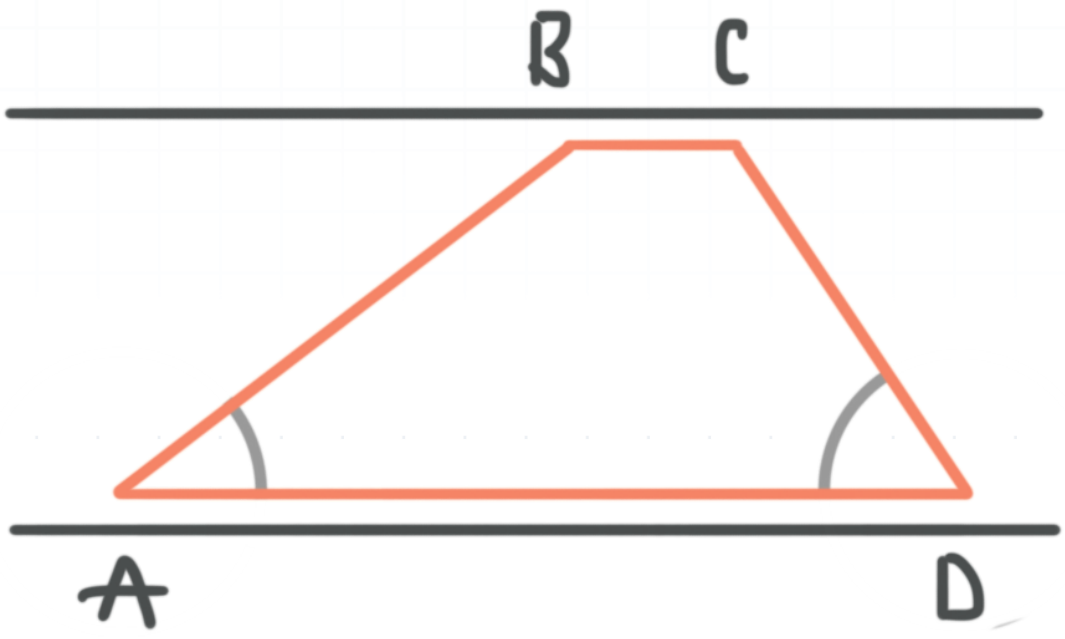
REFERENCE ANGLE THEOREM

■ 1. Sketch the graph of $R(x)$ for $x \in [0,2\pi]$, where the value of $R(x)$ is equal to the measure of its reference angle.

■ 2. A trapezoid-shaped plot of land lies between the two parallel railways. The land owner needs to know the sines of all four angles of the plot, and he already knows that the sines of two angles along the same railway are 0.27 and 0.46. Find the sines of the two remaining angles.



■ 3. Mickey wants to make a right kite (a kite with two opposite right angles). He has a piece of cloth for which he managed to measure the short sides, 26 cm, and the cosine of the smallest angle, 0.8. Find the width of the frame, which should join the right angles of the kite.

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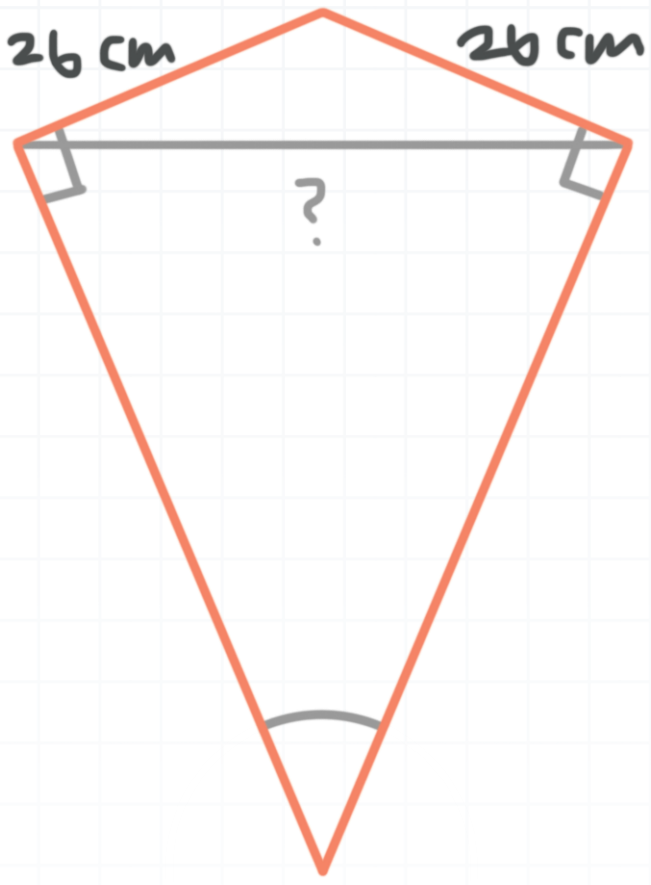

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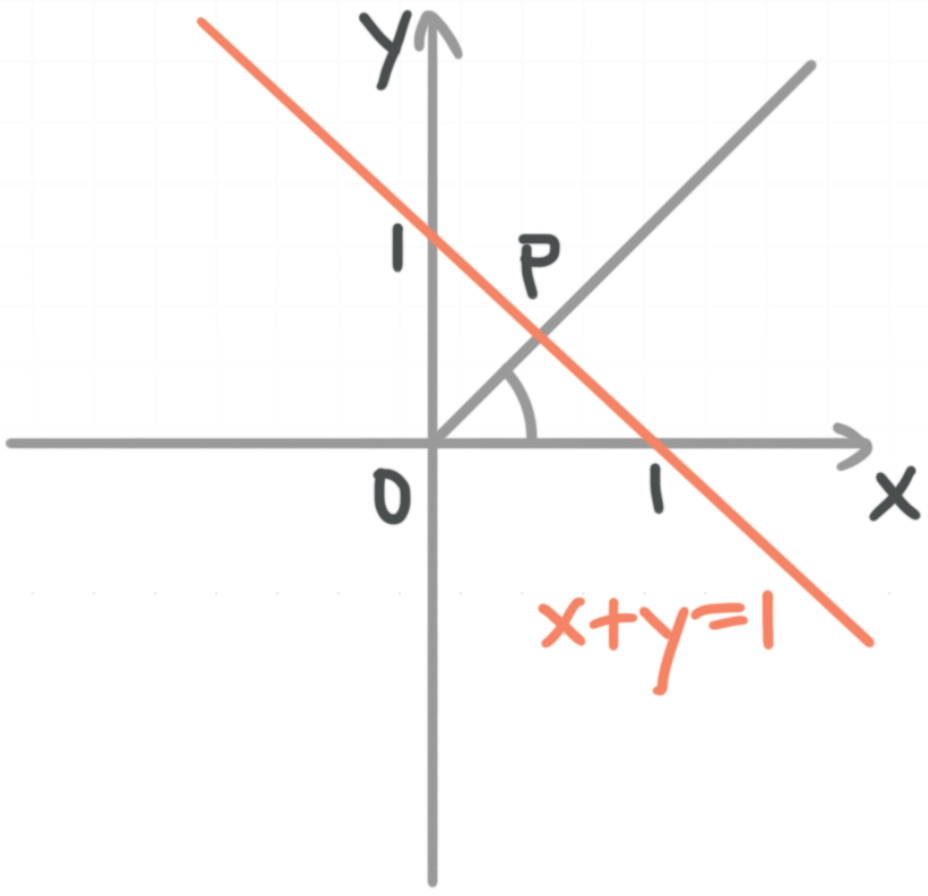
GIVEN A POINT ON THE TERMINAL SIDE OF THE ANGLE

■ 1. On the circle with center at the origin and radius 5, find the point(s) where $\cos\theta = -0.6$.

■ 2. Find the point P on the line $x + y = 1$ where sine of the angle between P and the positive direction of the x-axis is $\sqrt{2}/2$.



■ 3. An airplane flew 3 hours North-North-East (NNE) with a constant velocity of 120 km/h. How far north is the plane from the airport and how far east is the plane from the airport?

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